Magnetic Nanoparticle-Based Hyperthermia for Head & Neck Cancer in Mouse Models

Qun Zhao, et al.

Appendix: Mathematical Treatment of the simulation of heating experiment

We adopt Rosensweig's [22] model to calculate the heating power dissipation *P* generated by ferrofluids under alternating magnetic fields. We also use heat transfer equations for double layer spherical model described by Andra et al [29] and relevant thermal parameters provided by Sawyer et al [30]. Particles within ferrofluids are typically poly-dispersed. The size distribution of particles has a significant effect on magnetic heating power dissipation. We experimentally determined the median radius of magnetite particles in our ferrofluids to be 7.37 nm (R_0 = 7.37 nm), with a standard deviation of 0.05 (σ = 0.05). And the anisotropy constant of Fe_3O_4 is chosen to be 23 kJ/m^3.

First, we calculate the power density *P* of ferrofluids:

$$\bar{P} = \int_0^\infty p(R) g(R) dR$$

Where

$$p(R) = \pi \mu_0 \chi_0 H^2 f \frac{2\pi f \tau}{1+(2\pi f \tau)^2}, g(R) = \frac{1}{\sqrt{2\pi}\sigma R} exp\left(\frac{-(\ln(\frac{R}{R_0}))^2}{2\sigma^2}\right) \quad (1)$$

Where μ_0 is the permeability of vacuum, χ_0 is initial magnetic susceptibility of ferrofluids, *H* and *f* is the peak strength and frequency of alternating magnetic fields, τ is the relaxation time of magnetic particles.

The power density calculated above is the heating rate of pure magnetite. In a tumor, the power density of ferrofluid is:

$P = \phi\bar{P}$

where ϕ is the volume fraction of magnetite in tumor.

The heat transfer equations of double layer spherical model [29] are:

$$\rho_1 c_1 \frac{\partial T_1}{\partial t} = \frac{\lambda_1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T_1}{\partial r}\right) + P, when\ 0 \leq r < R_f \qquad (2)$$

$$\rho_2 c_2 \frac{\partial T_2}{\partial t} = \frac{\lambda_2}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T_2}{\partial r}\right), when\ r \geq R_f$$

Where ρ_1, c_1, λ_1, are the density, specific heat, and thermal conductivity of ferrofluids, and ρ_2, c_2, λ_2 are the density, specific heat, and thermal conductivity outside the ferrofluid region. R_f is the effective radius induced by injected ferrofluids. T_1 and T_2 is the temperature inside and outside the ferrofluid core.

With initial condition:

$$T(0, r) = T_0$$

and boundary conditions:

$$T(0, \infty) = T_0$$

$$T(t, R_f - \epsilon) = T(t, R_f + \epsilon)$$

Where, T_0 is assumed to be body temperature (310 K) as the initial temperature all over the simulation domain. And ϵ is an infinitely small value to describe the continuous constraint of temperature across the interface of double spherical layer.

Next, we apply Laplace transformation to above PDE system, an elementary solution can be obtained by using the inversion theorem according to reference [17].

$$\Delta T_1(r,t) = \frac{PR_f^2}{3\lambda_2}\left[1 + \frac{q_\lambda}{2}\left(1 - \frac{r^2}{R^2}\right) + \frac{6}{\pi} q_\lambda^{\frac{3}{2}} q^{\frac{1}{2}} \frac{R_f}{r}\int_0^\infty f(z;r,t) g_1(z;r) dz\right], when\ 0 \le r < R_f$$

$$\Delta T_2(r,t) = \frac{PR_f^3}{3\lambda_2 r}\left[1 + \frac{6q_\lambda}{\pi}\int_0^\infty f(z;r,t) g_2(z;r) \frac{dz}{z}\right], when\ R_f \le r \qquad (3)$$

Where,

$$q_\lambda = \frac{\lambda_2}{\lambda_1}, q = \frac{\rho_2 c_2}{\rho_1 c_1}, s(z) = (q_\lambda - 1)\sin z + z\cos z,$$

$$f(z;r,t) = z^{-2}\exp\left(-\frac{\lambda_1 t z^2}{\rho_1 c_1 R_f^2}\right)\frac{z\cos z - \sin z}{[s(z)]^2 + q_\lambda q (z\sin z)^2},$$

$$g_1(z;r) = \sin\left(\frac{rz}{R_f}\right), g_2(z;r) = s[z]\sin[k(z;r)] + (q_\lambda q)^{1/2} z \sin z \cos[k(z;r)],$$

$$k(z;r) = (q_\lambda q)^{\frac{1}{2}} z\left(\frac{r}{R_f} - 1\right)$$

The infinite integral terms in Equation (3) is evaluated by Laguerre Polynomial of degree 100. In the simulation, the following parameters of ferrofluids are used: ρ_1 = 1190 kg/m^3, c_1 = 1500 J/kg K and λ_1 = 0.5W/m K. Surrounding tumor tissues properties are: ρ_2 = 1000 kg/m^3, c_2 = 3500 J/kg K and λ_1 = 0.28W/m K.